CLASSES BY SACHIN SHARMA

DIFFERENTIATION - Exercise 5.7

CLASS 12TH - Ch-5

Exercise 5.7

–Second Part (Q11-17)

Here we need to do the differentiation of a function twice.

Q11: If y = 5 cosx -3 sin x, Prove that $\frac{d^2y}{dx^2} + y = 0$

Given : y = 5 cosx – 3 sinx ...(i)

$\therefore \frac{dy}{dx} = -5 \sin x - 3 \cos x$

Again differentiating w.r.t. x, $\frac{d^2y}{dx^2} = -5 \cos x + 3 \sin x$

$= -(5 \cos x - 3 \sin x) = -y$ By (i)

or $\frac{d^2y}{dx^2} = -y$

$\therefore \frac{d^2y}{dx^2} + y = 0$

Question 12:

If $y = \cos^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

Answer 12:

Given that: $y = \cos^{-1} x$,

$\Rightarrow \cos y = x$, therefore, $-\sin y \frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{\sin y} = -\operatorname{cosec} y$

$\Rightarrow \frac{d^2y}{dx^2} = -(\operatorname{cosec} y \cot y) . \frac{dy}{dx}$

$\Rightarrow = (\operatorname{cosec} y \cot y).(-\operatorname{cosec} y) = -\operatorname{cosec}^2 y \cot y$

Question 13:

If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2 y_2 + xy_1 + y = 0$

Answer 13:

$y = 3\cos(\log x) + 4\sin(\log x)$

Differentiating both sides w.r.t. X

$$\frac{dy}{dx} = -3\sin(\log x)\frac{1}{x} + 4\cos(\log x)\frac{1}{x}$$

$$X\frac{dy}{dx} = -3\sin(\log x) + 4\cos(\log x)$$

Again Differentiating both sides w.r.t. X

$$X\frac{d^2y}{dx^2} + \frac{dy}{dx} = -3\cos(\log x)\frac{1}{x} - 4\sin(\log x)\frac{1}{x}$$

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}= -3\cos(\log x)\frac{1}{x} - 4\sin(\log x)\frac{1}{x}$$

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}= -[3\cos(\log x)\frac{1}{x} + 4\sin(\log x)\frac{1}{x}]$$

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}= -y$$

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$$

Q14: If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

Given that: $y = Ae^{mx} + Be^{nx}$, therefore,

$$\frac{dy}{dx} = \frac{d}{dx}(Ae^{mx} + Be^{nx}) = mAe^{mx} + nBe^{nx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(mAe^{mx} + nBe^{nx}) = m^2Ae^{mx} + n^2Be^{nx}$$

Putting the value of $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ in $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$, we get

$$\begin{aligned}
LHS &= (m^2Ae^{mx} + n^2Be^{nx}) - (m+n)(mAe^{mx} + nBe^{nx}) + mny \\
&= m^2Ae^{mx} + n^2Be^{nx} - (m^2Ae^{mx} + mn^2Be^{nx} + mnAe^{mx} + n^2Be^{nx}) + mny \\
&= -(mnAe^{mx} + mn^2Be^{nx}) + mny \\
&= -mn(Ae^{mx} + Be^{nx}) + mny = -mny + mny = 0 = RHS
\end{aligned}$$

15. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$.

Sol.

Given: $y = 500e^{7x} + 600e^{-7x}$ ...(*i*)

$\therefore \quad \frac{dy}{dx} = 500e^{7x}(7) + 600e^{-7x}(-7)$

$= 500\,(7)e^{7x} - 600(7)e^{-7x}$

$\therefore \quad \frac{d^2y}{dx^2} = 500\,(7)e^{7x}(7) - 600(7)e^{-7x}(-7) = 500\,(49)e^{7x} + 600(49)e^{-7x}$

or $\frac{d^2y}{dx^2} = 49\left[500\,e^{7x} + 600\,e^{-7x}\right] = 49y$ By (*i*))

or $\frac{d^2y}{dx^2} = 49y$

Q16: If $e^y(x+1)=1$, show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$.

$e^y(x+1)=1,$

Differentiating both sides w.r.t.x

$e^y\frac{dy}{dx}(x+1)+(x+1)\frac{d}{dx}e^y=\frac{d}{dx}1$

$\Rightarrow e^y+(x+1)e^y\frac{dy}{dx}=0$

$e^y+e^y\frac{dy}{dx}x+e^y\frac{dy}{dx}=0$

$e^y+e^y\frac{dy}{dx}(x+1)=0$

$e^y\frac{dy}{dx}(x+1)=-e^y$

$\frac{dy}{dx}=-\frac{1}{x+1}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(-\frac{1}{x+1}\right)$$

$$\Rightarrow = -\left[\frac{(x+1).\frac{d}{dx}1 - 1.\frac{d}{dx}(x+1)}{(x+1)^2}\right]$$

$$\Rightarrow = -\left[\frac{0-1}{(x+1)^2}\right] = \frac{1}{(x+1)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \left(-\frac{1}{x+1}\right)^2 \qquad \Rightarrow \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

Question 17: If , show that $(x^2+1)^2 y_2 + 2x (x^2+1)^2 y_1 = 2$.

Answer 17:

$y = (\tan^{-1}x)^2$ $\qquad \frac{dy}{dx} = \frac{2\tan^{-1}x}{1+x^2}$

$\Rightarrow (1+x^2)\frac{dy}{dx} = 2\tan^{-1}x$

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{d}{dx}\left(1+x^2\right) = \frac{d}{dx}(2\tan^{-1}x)$

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 2x = \frac{2}{1+x^2}$

$\Rightarrow (1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2$ $\qquad (x^2+1)^2 y_2 + 2x (x^2+1)^2 y_1 = 2$